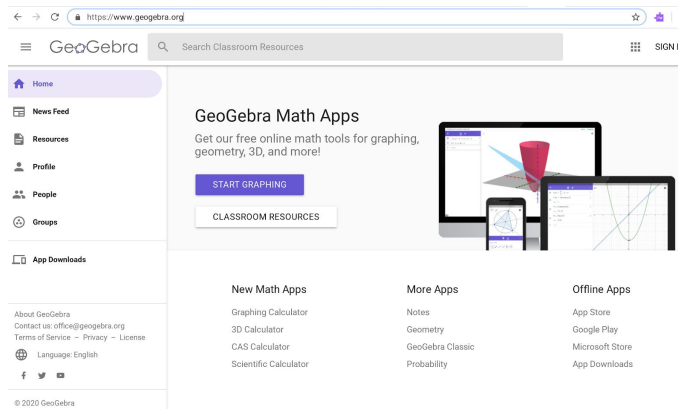
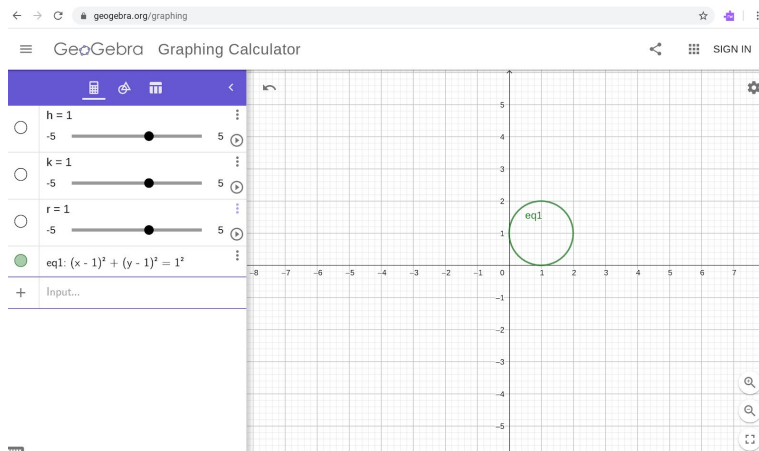


## Equation of a Circle

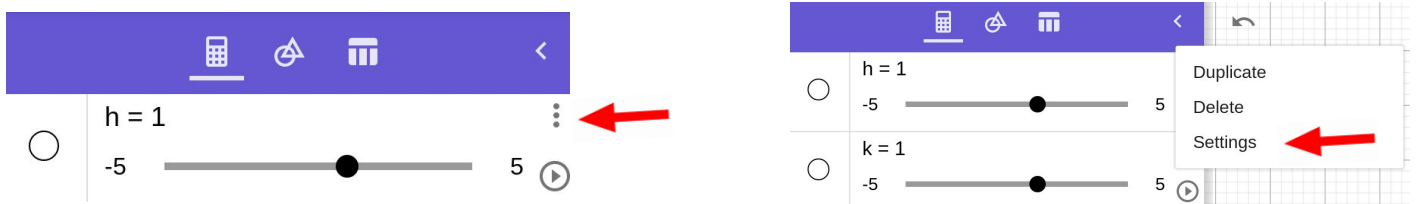
1. The technology that we will be using today is GeoGebra. Go ahead and open up the GeoGebra site using this link: <https://www.geogebra.org/>. After reaching the home page, click on “START GRAPHING” so we can begin this activity.



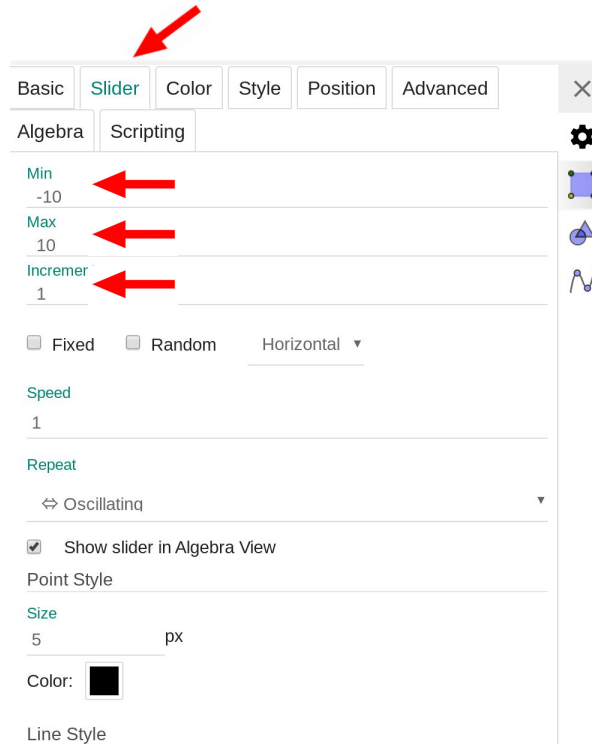
2. In the first input section, type in  $(x - h)^2 + (y - k)^2 = r^2$ , which is the center-radius form of the general equation for any circle with radius  $r$  and center  $(h, k)$ , then press enter. (Since we entered an equation with undefined variables, the site provided us with sliders that we can use to adjust the different variables in the equation.)



3. Go ahead and click the three dots on the top right corner of the first input box containing the variable  $h$ . Three options will appear and you can click on settings.



4. A settings box should appear on the right side of your screen. Click on the tab that says “slider” so you can change the min value to -10, the max value to 10, and the increment to 1. After you enter these numbers in, hit enter. Repeat this for the variable  $k$ . For the variable  $r$ , change the min value to -20, the max value to 20, and the increment to 1 as well. Adjusting the sliders will allow us to have a wider range to create a larger circle. You can then close the settings box.

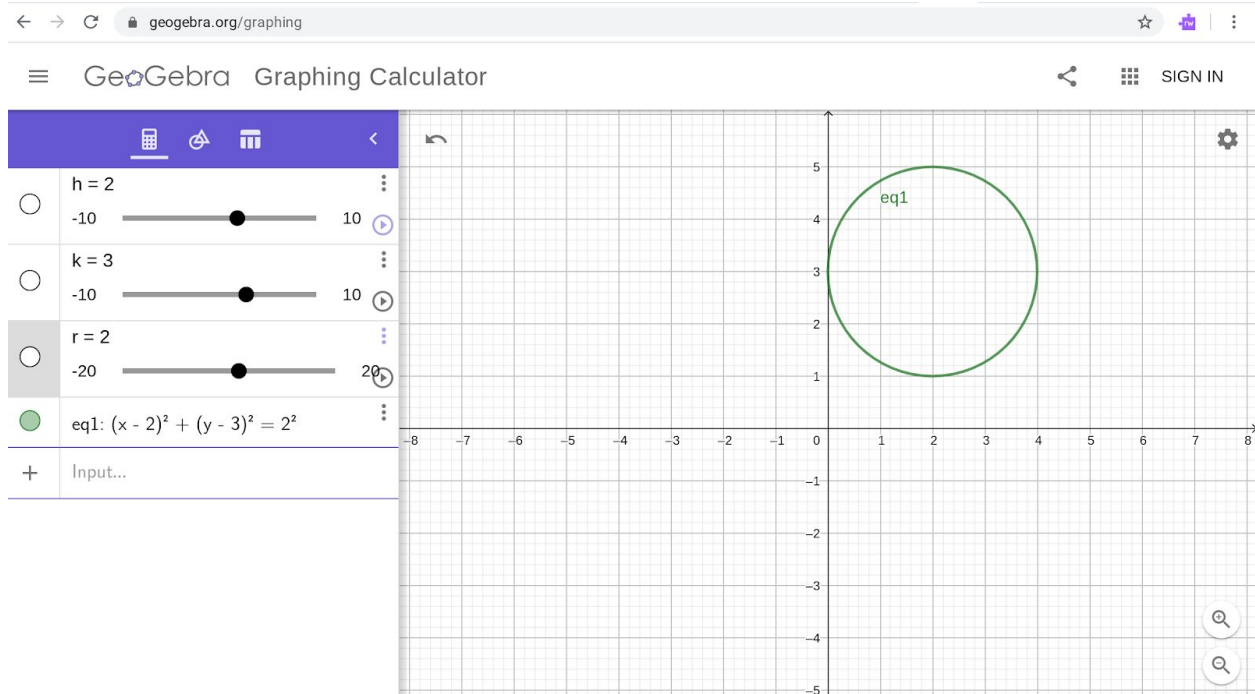


5. Now you can play around with the sliders. What happens when you change the variable  $h$ ? What happens when you change the variable  $k$ ? What about  $r$ ?

### Examples to Try

1. Reminder: The standard form of the equation of a circle is  $(x - h)^2 + (y - k)^2 = r^2$ , where:
  - a.  $r$  is equal to the radius of a circle.
  - b.  $h$  and  $k$  are the coordinates of the center of a circle.
2. **Example:** Find the center and radius of  $(x-2)^2 + (y-3)^2 = 4$ 
  - a. The center of this circle is (2,3).

- i. A common mistake is to make  $h = -2$  and  $k = -3$ . In this equation, the sign preceding  $h$  and  $k$  are negative, so  $h$  and  $k$  are positive values. Therefore,  $h = 2$  and  $k = 3$ .
- b. The radius of this circle is 2, because the  $\sqrt{4} = 2$ .
- c. We can check our answer using GeoGebra. Using the sliders, we can set  $h$  equal to 2,  $k$  equal to 3 and  $r$  equal to 2.



### General Form of the Equation

1. If the equation is in the general form,  $x^2 + y^2 + Dx + Ey + F = 0$  where  $D, E, F$  are constants, we have to complete the square and turn the equation into its standard form. Then we can find the center and radius easily.
2. **Example:**  $x^2 + y^2 - 6x + 4y + 9 = 0$ 
  - a. Completing the square:

- i. Write the equation in this form:

$$(x^2 - 6x + ?_1) + (y^2 + 4y + ?_2) = -9 + ?_1 + ?_2$$

In the first parenthesis, we group the  $x$ -terms and in the second the

y-terms. The constant is moved on the right hand side. The question mark, ?, is the number needed in each parenthesis to complete the square. Note that we have to add this number to both sides of the equation.

- ii. How to find the number to replace the question mark, ?<sub>1</sub>. Take the coefficient of x and divide it by 2, (-6/2), and then square it, (-3)<sup>2</sup> = 9. ?<sub>1</sub> is going to be replaced by the number 9.
- iii. How to find the number to replace the question mark, ?<sub>2</sub>. Take the coefficient of y and divide it by 2, (4/2), and then square it, (2)<sup>2</sup> = 4. ?<sub>2</sub> is going to be replaced by the number 4.
- iv. Now if we put all of this information together, we get:

$$(x^2 - 6x + ?_1) + (y^2 + 4y + ?_2) = -9 + ?_1 + ?_2$$

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = -9 + 9 + 4$$

$$(x - 3)^2 + (y + 2)^2 = 4$$

$$(x - 3)^2 + (y - (-2))^2 = 4$$

This equation is now in the standard form. The center is (3,-2) and the radius is 2. We can check this by plugging in the general form of the equation into GeoGebra.

